

Stratified Programs

Observation:

Every normal program is consistent (has a model), but this is not necessarily true for $\text{comp}(P)$.

Example:

program P:

$p \leftarrow \sim q$

$q \leftarrow \sim r$

$r \leftarrow \sim p$

$I = \{p, q, r\}$ is a model

$\text{comp}(P)$

$p \leftrightarrow \sim q$

$q \leftrightarrow \sim r$

$r \leftrightarrow \sim p$

$p \leftrightarrow \sim q \leftrightarrow r \leftrightarrow \sim p$

By transitivity:

$p \leftrightarrow \sim p$

Thus there exists no model for $\text{comp}(P)$

Limit the use of negation in recursion

Definition:

A *level mapping* of a normal program is a mapping from its set of predicate symbols to the non-negative integers. We refer to the value of a predicate symbol under this mapping as the *level* of that predicate symbol.

Definition:

A normal program is hierarchical if it has a mapping such that in every program clause

$A \leftarrow L_1, \dots, L_n$, the level of every predicate symbol occurring in the body is less than the level of A .

Observation:

not hierarchical:

$\text{relatedTo}(x,y) \leftarrow \text{relatedTo}(y,x)$

Definition:

A normal program is stratified if it has a level mapping such that in every clause $A \leftarrow L_1, \dots, L_n$,

- the level of the predicate symbol of every positive literal is less or equal to the level of A and
- the level of each predicate symbol of every negative literal is less than the level of A .

level 2

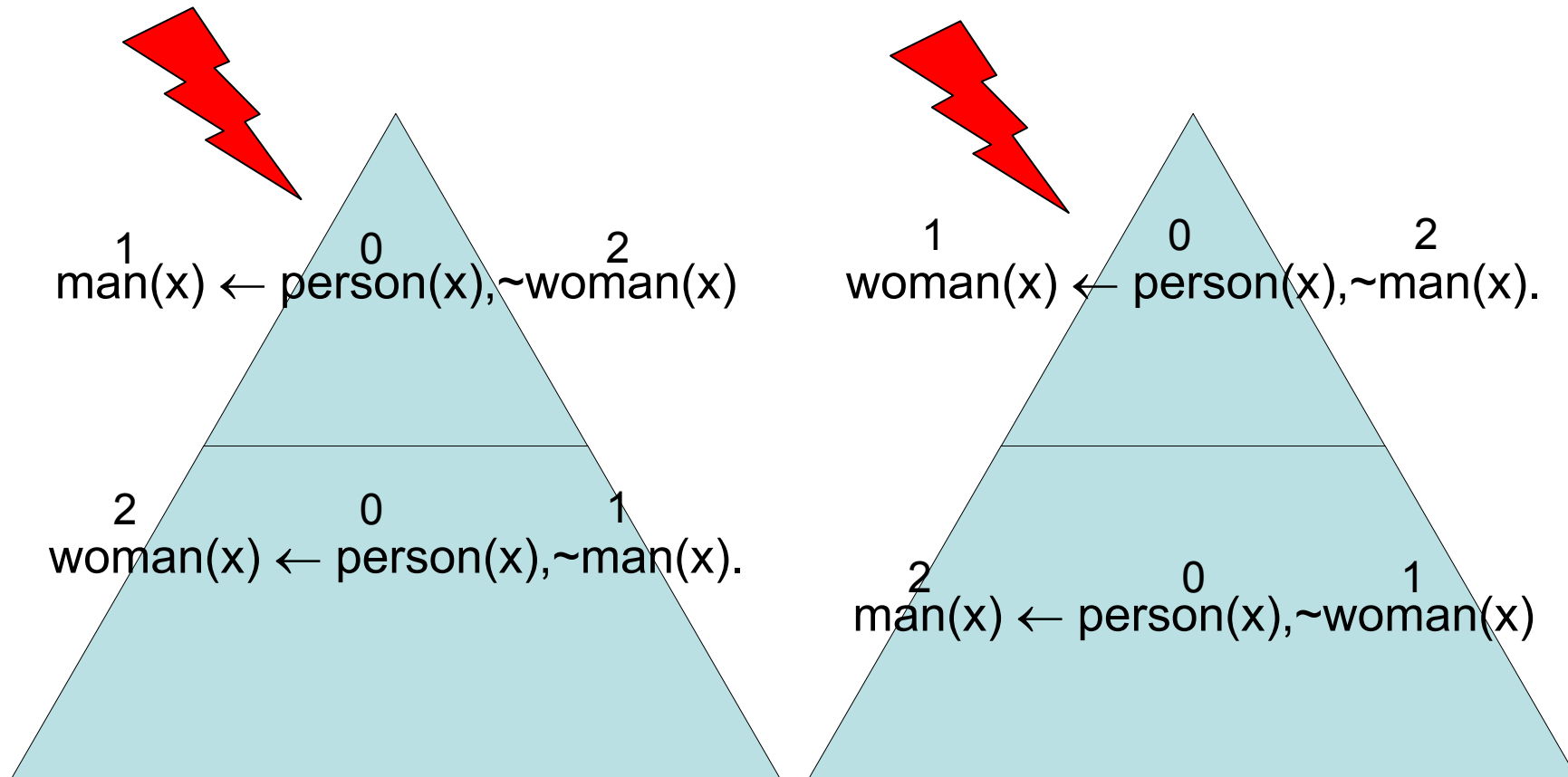
$\text{loves}(x,y) \leftarrow \text{friend}(x,y)$
 $\text{loves}(x,y) \leftarrow \text{enemy}(x,y)$

level 1

$\text{enemy}(x,y) \leftarrow \sim \text{friend}(x,y)$

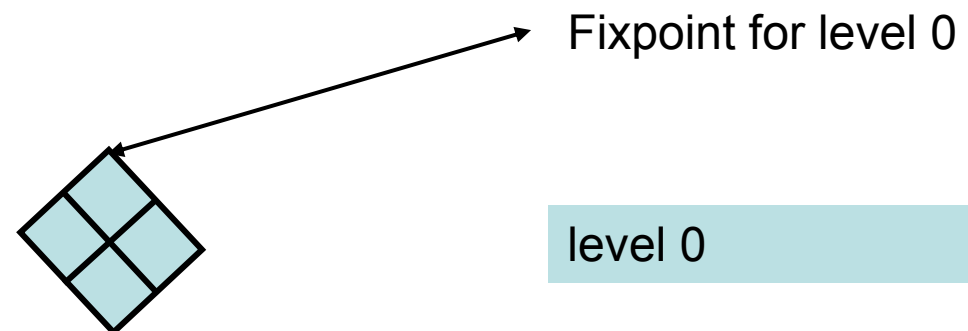
level 0

$\text{friend}(x,z) \leftarrow \text{friend}(x,y), \text{friend}(y,z)$
 $\text{friend}(a,b) \leftarrow$
 $\text{friend}(b,c) \leftarrow$



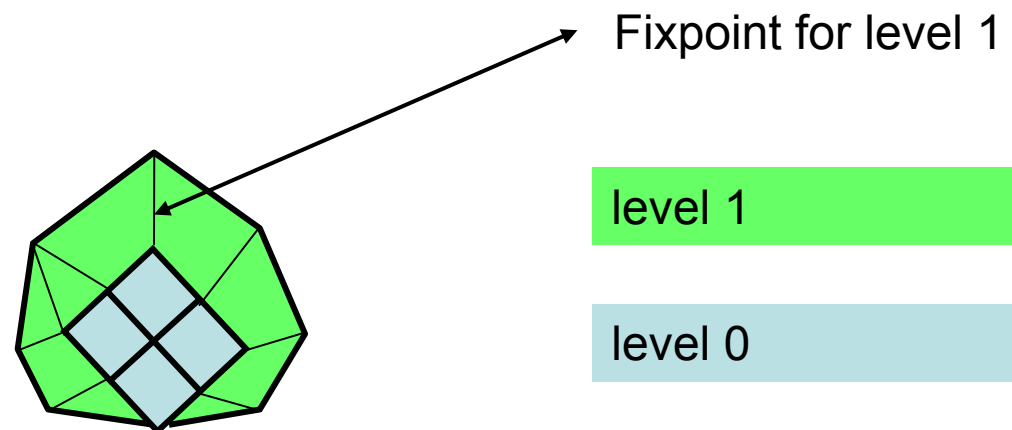
Corollary:

Let P be a stratified normal program. then $\text{comp}(P)$ has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to „=“.



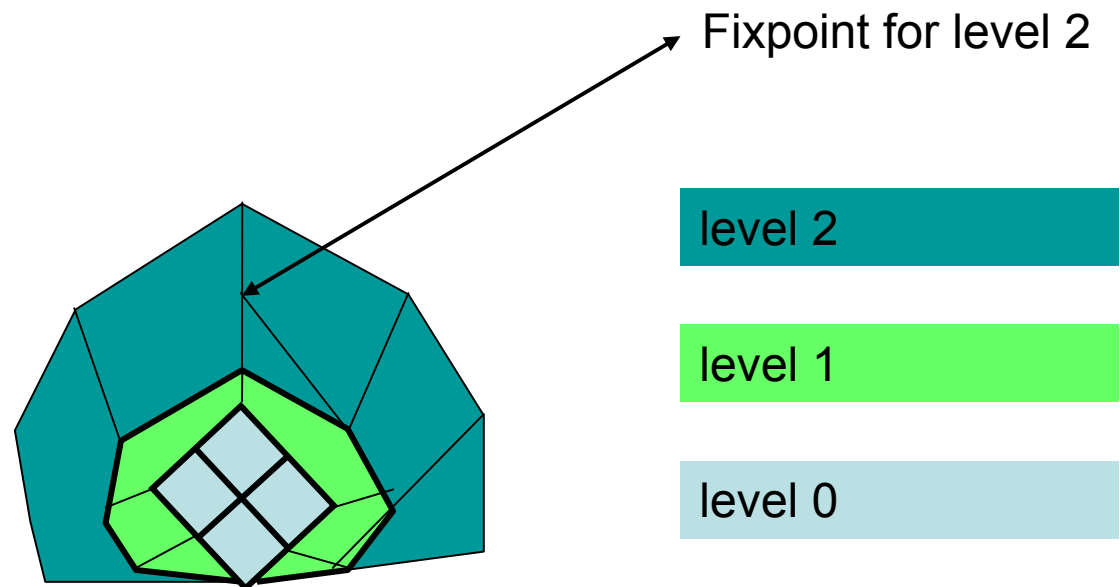
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Let P be a stratified normal program. then $\text{comp}(P)$ has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to „=“.



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Definition

A normal program is locally stratified if each atom in B_P can be assigned a countable ordinal level such that no atom

- positively depends of an atom of greater level
- negatively depends of an atom of equal or greater level.

$\text{even}(s(X)) \leftarrow \neg \text{even}(X).$
 $\text{even}(0).$

$B_P:$

$\{\text{even}(0)^0, \text{even}(s(0))^1, \text{even}(s(s(0)))^2,$
 $\text{even}(\dots)^3, \dots\}$

Definition

Let P be a normal program and I a model. I is a perfect model for a given level of B_P , if
for every other model J ,
if a positive literal p is the atom of least level in one model, but not in the other, then p is in J .

In other words, atoms of higher level are preferred for the perfect model.

Przymusinski: All locally stratified programs have a perfect model, which is independent of the ranking system chosen.

$\text{even}(s(X)) \leftarrow \neg \text{even}(X).$

$\text{even}(0).$

$\text{even}(0) \leftarrow q(X).$

$B_P:$

$J = \{q(0)^0, \text{even}(0)^1, \text{even}(s(s(0)))^3, \dots\}$

$I = \{\text{even}(0)^0, \text{even}(s(s(0)))^2, \dots\}$