Some Examples of Semi-rational DAG Languages

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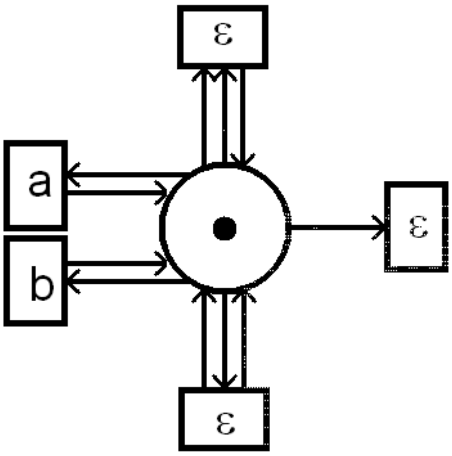
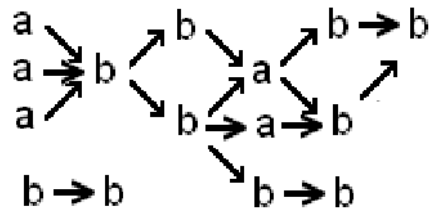
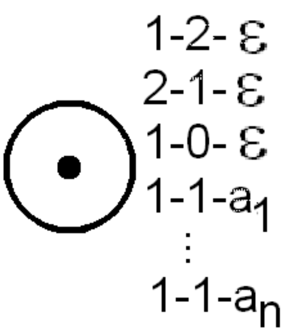


$\{a,b\}^\dagger$

Ex.:



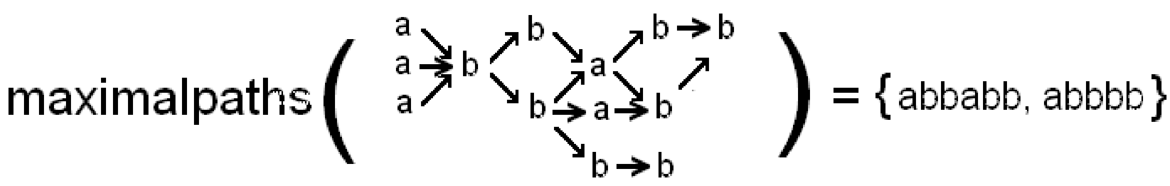


$\{a_1, \ldots, a_n\}^\dagger$

The *fat embedding* $D : 2^{\Sigma^*} \to 2^{\Sigma^\dagger}$:

for $L \subseteq \Sigma^*$ set

$$D_L := \{\alpha \in \Sigma^\dagger | maximalpaths(\alpha) \subseteq L\}.$$



Theorem
L rational $\Rightarrow$ D_L semi-rational.

Conjecture
D_L semi-rational $\Rightarrow$ L rational.

Theorem

$\Sigma^{\dagger}_{roots=i}$, $\Sigma^{\dagger}_{leaves=i}$, $\Sigma^{\dagger}_{nodes=i}$,

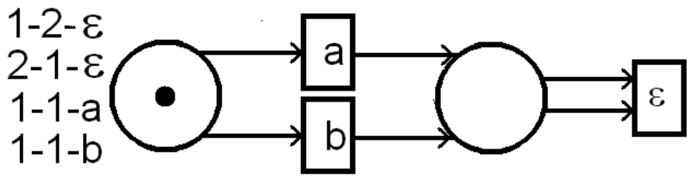
$\Sigma^{\dagger}_{r\,mod\,i}$, $\Sigma^{\dagger}_{l\,mod\,i}$, $\Sigma^{\dagger}_{n\,mod\,i}$,

$\Sigma^{\dagger}_{sons\neq}$, $\Sigma^{\dagger}_{fathers\neq}$, $\Sigma^{\dagger}_{sons=}$, $\Sigma^{\dagger}_{fathers=}$,

are semi-rational.

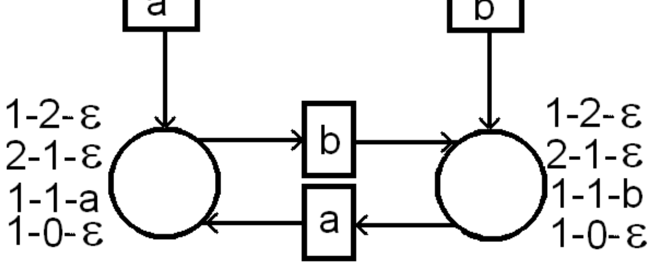
$\{a,b\}^{\dagger}_{\text{leaves mod 2}}$

$\{\alpha|\#$ leaves is even$\}$



$\{a,b\}^{\dagger}_{\text{fathers=}}$

$\{\alpha|$ all nodes possess only father-nodes of the same label$\}$



Theorem The class of semi-rational *dag* languages is closed under
$\bigcup$, $\bigcap$, $\circ$, $\|$, ^{R},
but not under complement, * , $\|^*$.

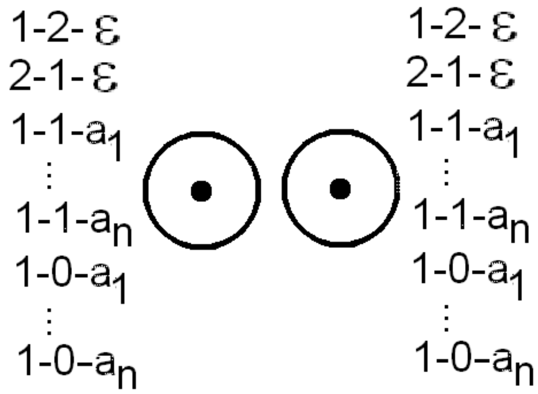
An *undirected* path in a dag is a path that may pass an arc forward or backward.

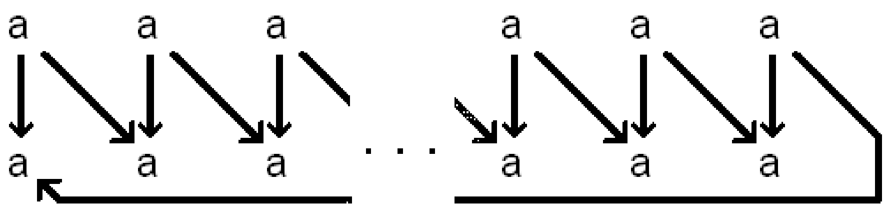
connected dag : any two nodes are connected by an undirected path,
otherwise : *disconnected*.

$\Sigma^\dagger_{con}$ is the language of all connected *dag*s,
$\Sigma^\dagger_{dis}$ of all disconnected *dag*s over Σ,

Theorem $\Sigma^\dagger_{dis}$ is semi-rational, $\Sigma^\dagger_{con}$ is not.

$\{a_1, \ldots, a_n\}^{\dagger}_{dis}$

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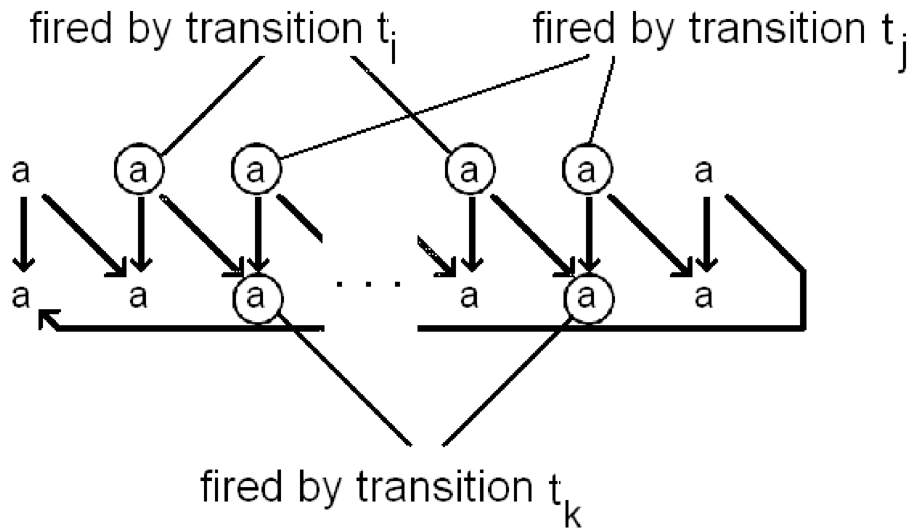
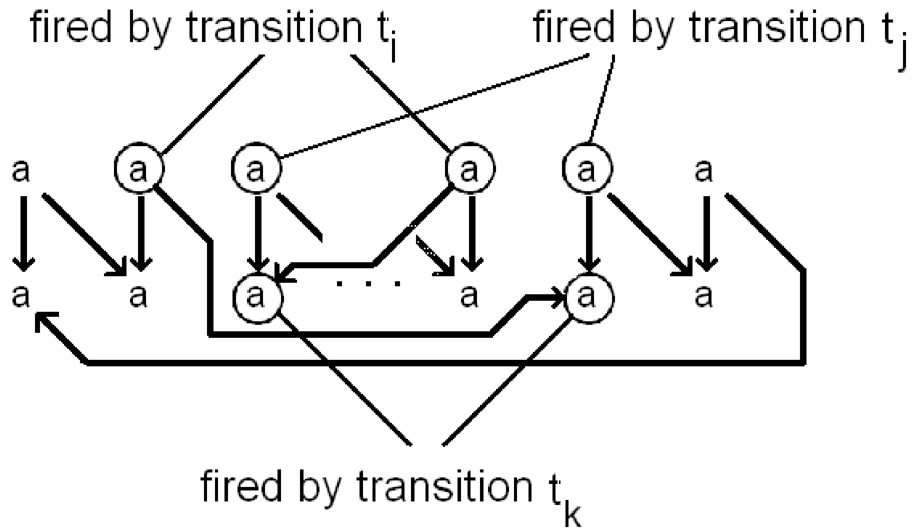
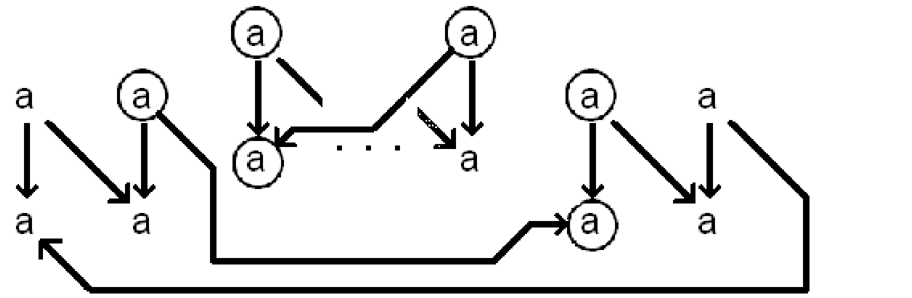
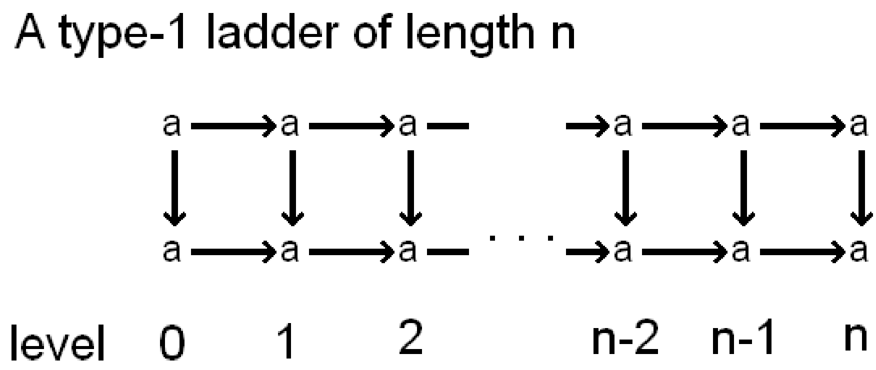

fired by transition t_i
fired by transition t_j
fired by transition t_k


fired by transition t_i
fired by transition t_j
fired by transition t_k





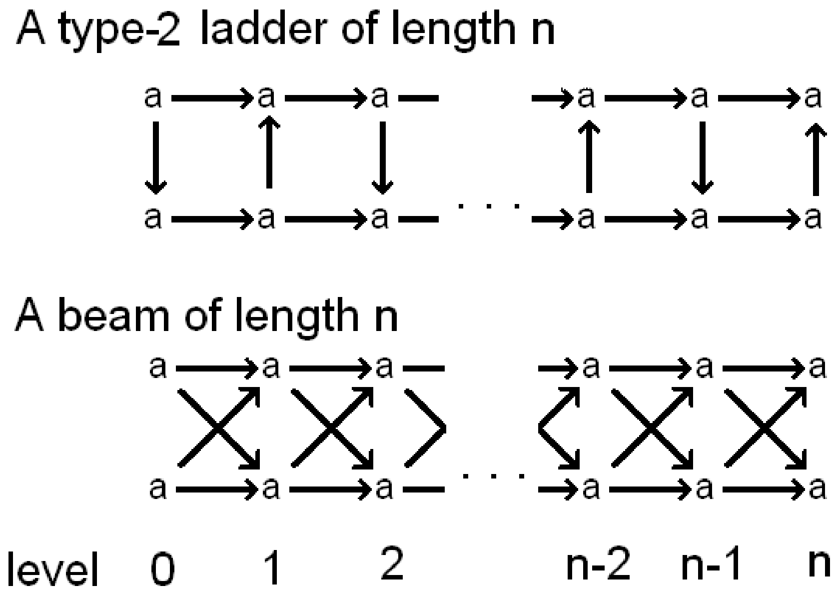
The same technique shows:

Theorem

$\{\text{type1-ladder}(n) | n \in \mathbb{N}\}$ is not semi-rational,

$\|^* \Box$ is not semi-rational.

where $\Box$ is the type-1-ladder of length 1.



However:

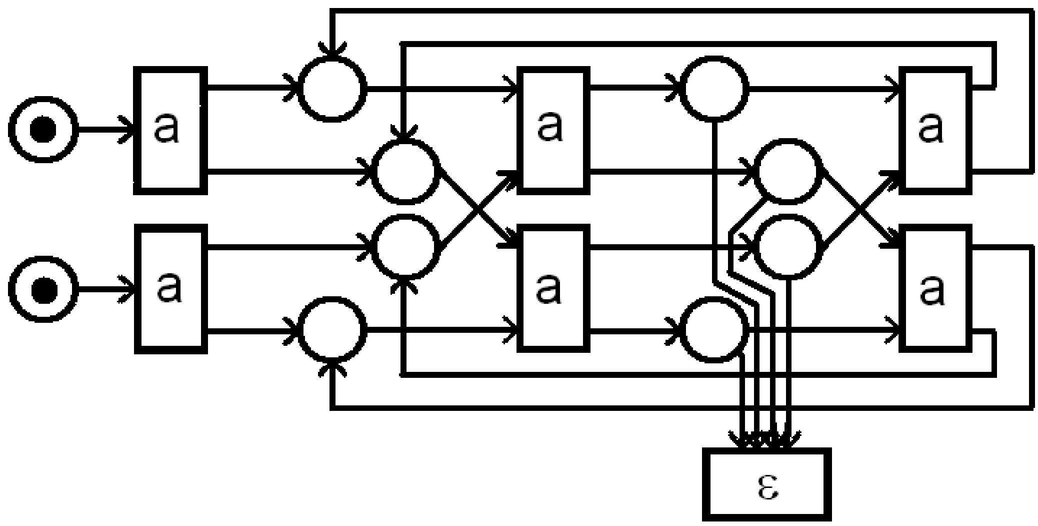
Theorem The following are semi-rational:

$\{$type2-ladder$(n) | n \in \mathbb{N}\}$,

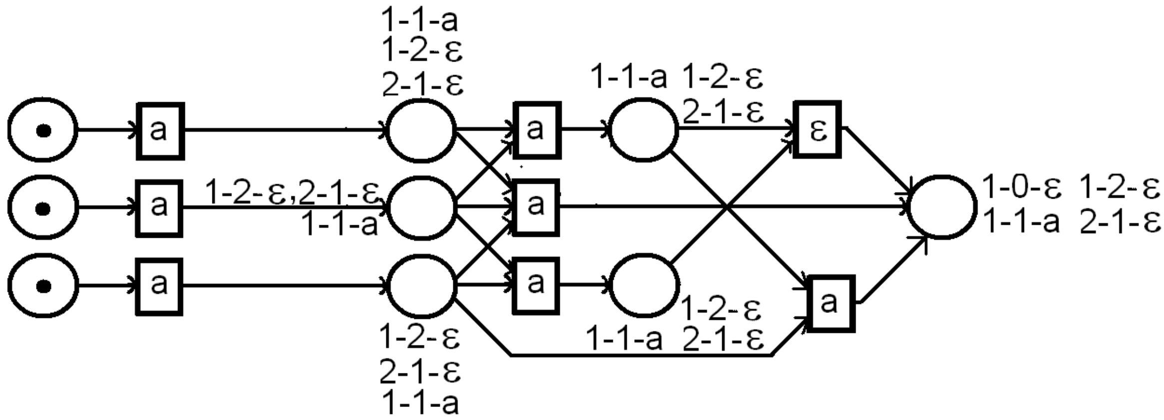
$\{$beam$(n) | n \in \mathbb{N}\}$,

$\Sigma^{\dagger}_{con,roots=i}$.

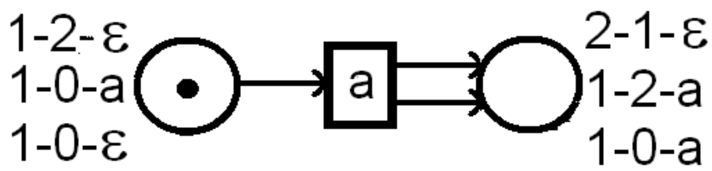
$\mathcal{N}_{\text{beam}}$



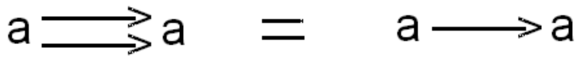
$\mathcal{N}_{\text{con, roots=3}}$



Attempt to generate $\{a\}^{\dagger}_{outdegree=2}$:



However, this net generates also



A 'better' (?) true-concurrency semantics: *dags* with multiple arcs.

Theorem

$\Sigma^{\dagger}_{indegree \leq i, outdegree \leq j}$
$\Sigma^{\dagger}_{indegree \leq 1, outdegree \, mod \, j}$

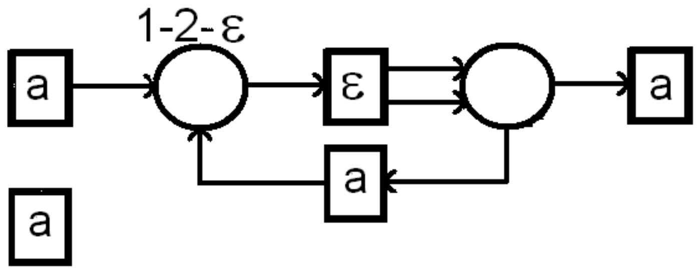
are semi-rational.

But:

Conjecture

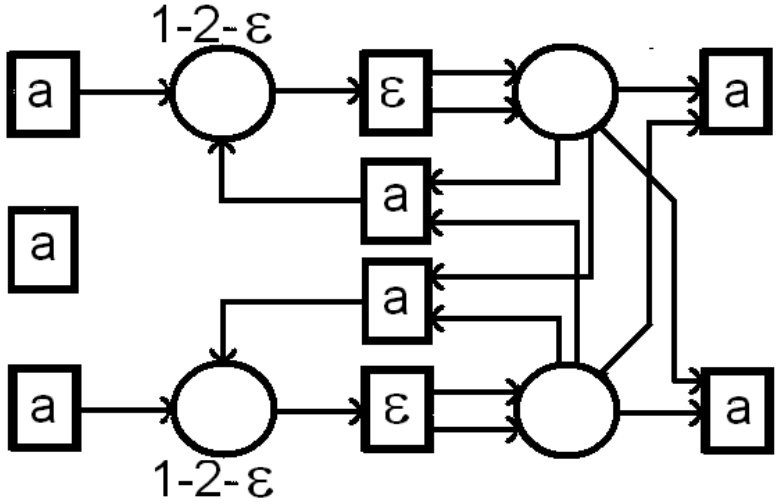
$\Sigma^{\dagger}_{indegree = c, outdegree \, mod \, j}$, with $c > 1$,
$\Sigma^{\dagger}_{indegree = (\leq) i}$, $\Sigma^{\dagger}_{outdegree = (\leq) i}$,
$\Sigma^{\dagger}_{indegree \, mod \, i}$, $\Sigma^{\dagger}_{outdegree \, mod \, i}$

are not semi-rational.



generates $\{a\}^{\dagger}_{indegree=1, outdegree\, mod\, 2}$.

Attempt to generate $\{a\}^{\dagger}_{indegree=2, outdegree\, mod\, 2}$:



but this net cannot generate